Functional Analysis 2020-2021

Syllabus

1. Linear operators:
- Bounded linear operators, adjoints and their properties. Projections in Hilbert spaces,
Neumann series.
- Compact operators: definition, examples and main properties (composition, closedness,
adjoint).
- Fredholm theory: statement and proof.

2. Spectral theory:
- Definition and topological properties of the spectrum, decomposition of the spectrum,
examples (Volterra, right and left shift, multiplication operators on sequences spaces).
- Spectrum of selfadjoint operators: reality, Weyl ‘s criterion, extremal of the spectrum.
- Spectrum of compact operators: main properties, spectral theorem for compact selfadjoint
operators, variational method to compute eigenvalues, application to integral operators.

3. Spectral theorem:
- Continuous functional calculus: spectral mapping theorem, isometric property, proof of the
spectral theorem
- Borellian functional calculus: the spectral measure, construction of bounded Borel functional
calculus and its properties.
- Spectral theorem for bounded linear selfadjoint operators: projection valued measure and
their property, spectral intergrals and its functional calculus properties, proof of the spectral
theorem.
- Applications: characterization of the spectrum, dynamics of Schroedinger equation

4. Sturm-Liouville problems
- Weak formulation of Sturm-Liouville problems; construction of weak solutions via Lax-
Milgram in case of small potential; compactness of the solution map; extension to large

potentials via Fredholm theory; return to classical solutions; Neumann boundary problem.
- Spectral analysis of Sturm-Liouville problems.

5. Differential calculus in Banach spaces
- Frechet , Gateux derivatives and their relations; higher order differentiability and Taylor
formula.
-Nemitski operator: continuity and differentiability over continuous and L∧p integrable
spaces.
-Implicit function theorem in Banach spaces, inverse function theorem.
-Applications to semilinear Sturm-Liouville problem and construction of periodic solutions of ODEs
-Lagrange multipliers: Hilbert and Banach case, complementary spaces, application to Sturm-
Liouville problems.

6. Introduction to bifurcation theory
-Lyapunov-Schmidt reduction.
-Crandall-Rabinowitz theorem of bifurcation from simple eigenvalues.
-Applications to nonlinear Sturm-Liouville problems.
-Stokes wave for water waves.

7. Schauder fixed point theorem
- Nonlinear compact maps.
- Schauder's theorem and applications.